QUANTUM COMPUTING AND ERROR CORRECTION

QUANTUM COMPUTING BENCHMARKING

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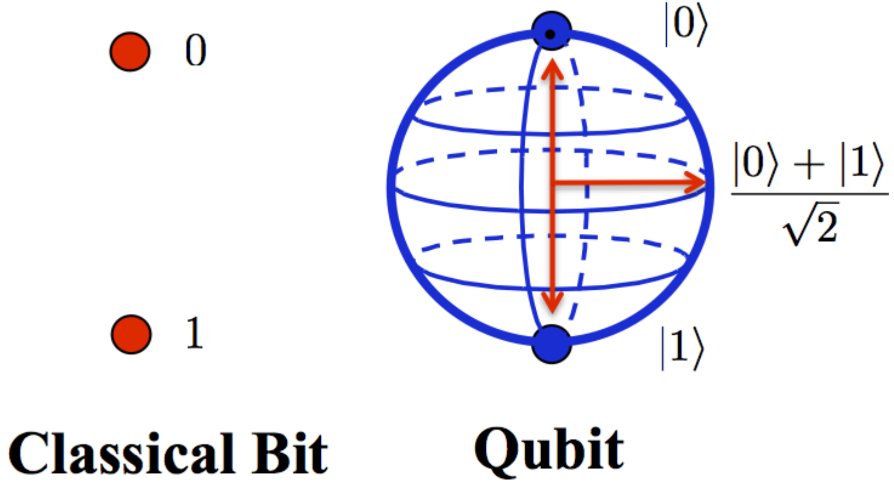
OVERVIEW

1. Introduction to qubits and quantum computing
    1. Classical bit vs. qubit
    2. Superposition states, quantum measurement
    3. Entanglement
    4. No-cloning theorem
2. Quantum errors
3. Quantum error correction
    1. Bit flip errors
    2. Phase errors
    3. Collective measurement and fault tolerance
4. Quantum error correction codes
    1. Surface codes, general LDPC codes
5. Current technologies + future work

INTRO TO QUBITS AND QUANTUM COMPUTING

CLASSICAL BIT VS. QUBIT



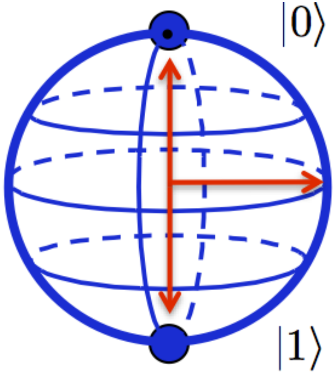
poetryinphysics.wordpress.com/

Classical bit: binary 0 or 1 state

Qubit (quantum bit):
superposition of 0 and 1 states
(0 and 1 denoted $|0\rangle$ and $|1\rangle$)

SUPERPOSITION STATES

General qubit superposition state: $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$



$|0\rangle \rightarrow a = 1,\ b = 0$

$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \rightarrow a = b = \frac{1}{\sqrt{2}} \rightarrow$ Equal superposition

$|1\rangle \rightarrow a = 0,\ b = 1$

Qubit

poetryinphysics.wordpress.com/

QUANTUM MEASUREMENT

- General qubit superposition state: $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$

- a and b are *probability amplitudes*:
  - Probability of measuring $|0\rangle$: $P_0 = |a|^2$
  - Probability of measuring $|1\rangle$: $P_1 = |b|^2$
  - Normalization: $P_0 + P_1 = 1$

- For $|\psi\rangle = a\,|0\rangle + b\,|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$

$$P_0 = P_1 = \frac{1}{2}.$$

→ Equal superposition

MULTI-QUBIT STATES

- Possible outcomes of measuring two qubits:

1st qubit → $|0\rangle\,|0\rangle$ ← 2nd qubit $\qquad |0\rangle\,|1\rangle$

$|1\rangle\,|0\rangle \qquad |1\rangle\,|1\rangle$

(denote $|0\rangle\,|0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |00\rangle$)

- General two-qubit state: $|\psi\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle$

Can extrapolate to *n* qubits

ENTANGLEMENT

Two qubits are entangled when their collective state cannot be separated into states of each individual particle.

E.g. Not entangled: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle = \frac{1}{\sqrt{2}}|0\rangle(|0\rangle + |1\rangle)$

Entangled: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

ENTANGLEMENT, CONT.

When two qubits are entangled, measuring one gives information about (aka determines the state of) the other.

E.g. $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$P_{00} = P_{11} = (\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$: 50% of the time $|00\rangle$, 50% of the time $|11\rangle$

If measure blue qubit as $|0\rangle$, state must be $|00\rangle$ and red qubit is in $|0\rangle$.
If measure blue qubit as $|1\rangle$, state must be $|11\rangle$ and red qubit is in $|1\rangle$.
Etc.

NO-CLONING THEOREM

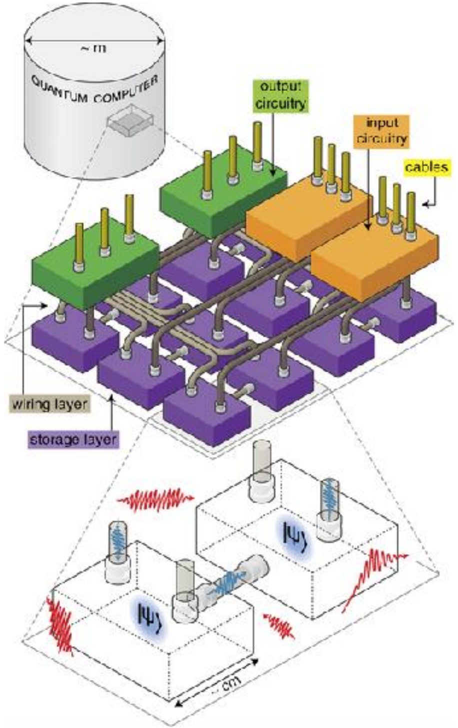
"It is impossible to create an independent and identical copy of an arbitrary unknown quantum state" (Wikipedia).

I.e. there is no unitary *U* such that

$$U(|A\rangle \otimes |B\rangle) = |A\rangle \otimes |A\rangle$$

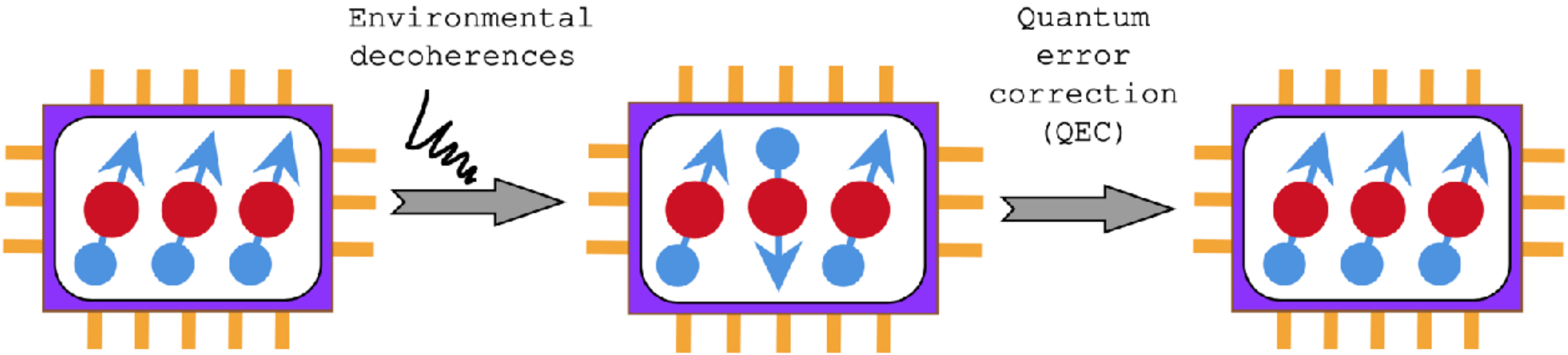
QUANTUM ERRORS

QUANTUM COMPUTER LAYOUT



Pfaff et al., 2015

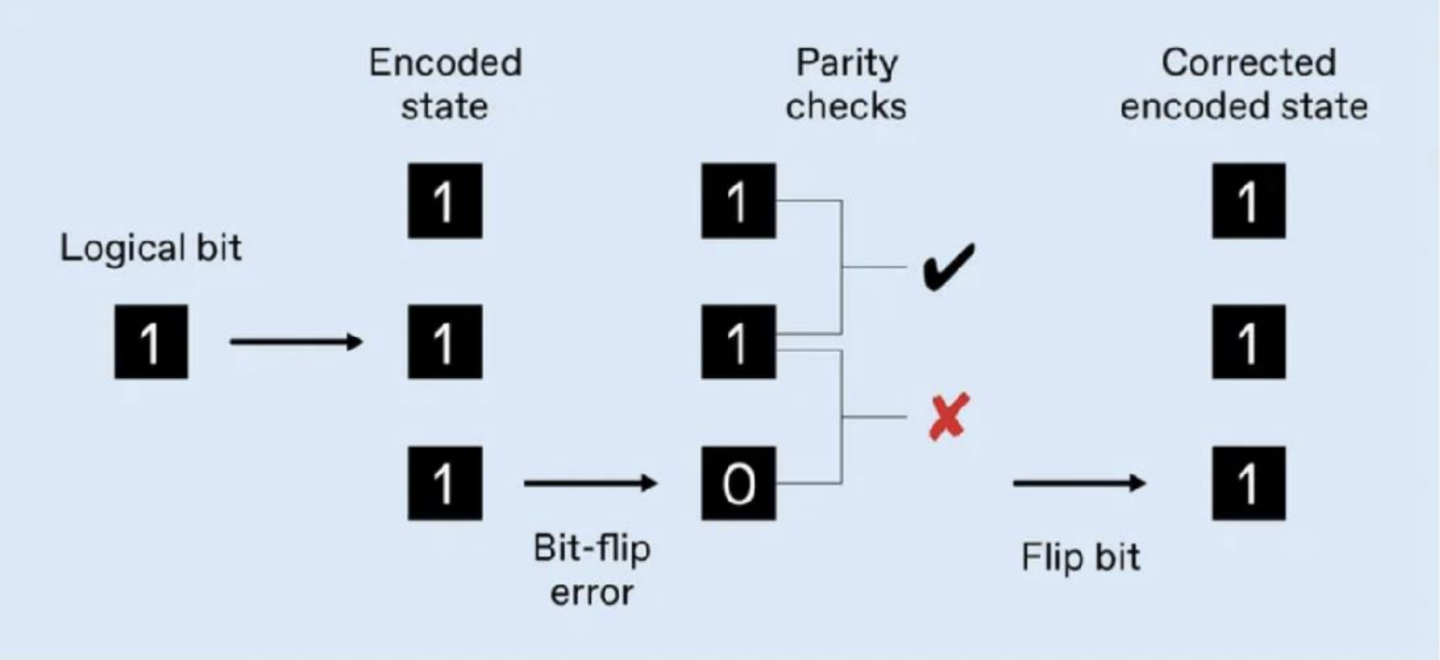
DECOHERENCE



Borah, 2022

CLASSICAL ERROR CORRECTION

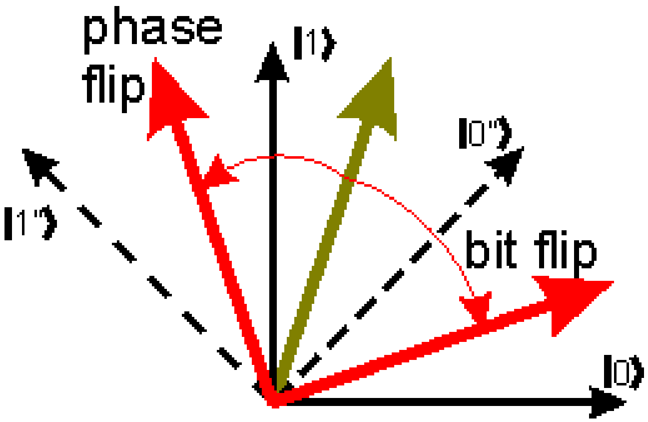
https://spectrum.ieee.org/quantum-error-correction



Biercuk and Stace, 2022

CHALLENGES FOR QUANTUM ERROR CORRECTION

- No-cloning theorem
- Different kinds of error
  - Bit-flip error
  - Phase error
- Probability continuum of errors
- Destructive measurement
  - $a\,|01\rangle + b\,|10\rangle$



[Steane, Quantum Error Correction, *an informal introduction*]

QUANTUM ERROR CORRECTION: BIT-FLIP ERRORS

BIT-FLIP ERROR

Bit-flip error: $a|0\rangle + b|1\rangle \rightarrow a|1\rangle + b|0\rangle$

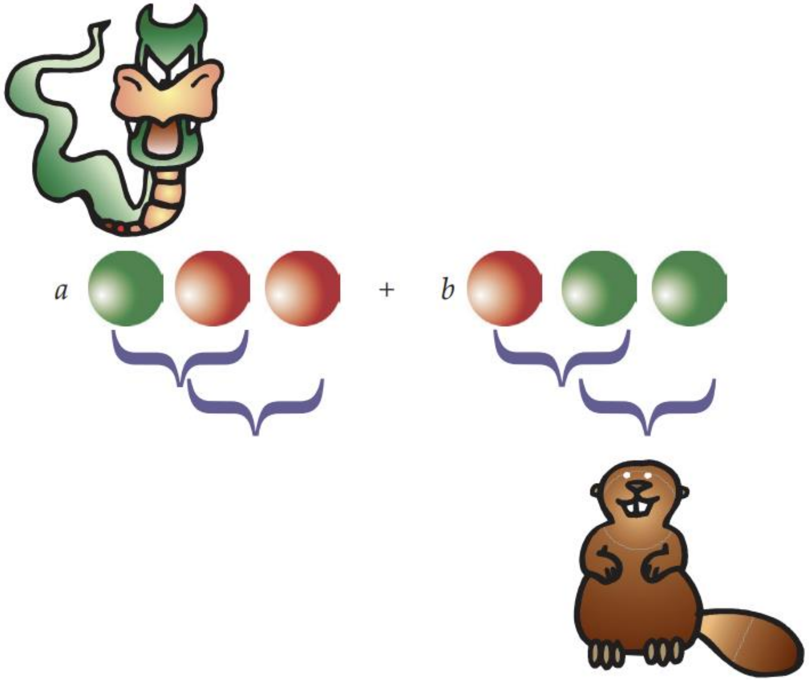
$$|0\rangle \rightarrow |\bar{0}\rangle \equiv |000\rangle$$

$$|1\rangle \rightarrow |\bar{1}\rangle \equiv |111\rangle$$

$$\text{General state: } a|0\rangle + b|1\rangle \rightarrow a|\bar{0}\rangle + b|\bar{1}\rangle \equiv a|000\rangle + b|111\rangle$$

$$\text{Bit flip: } a|000\rangle + b|111\rangle \rightarrow a|100\rangle + b|011\rangle$$

CORRECTING BIT-FLIP ERRORS



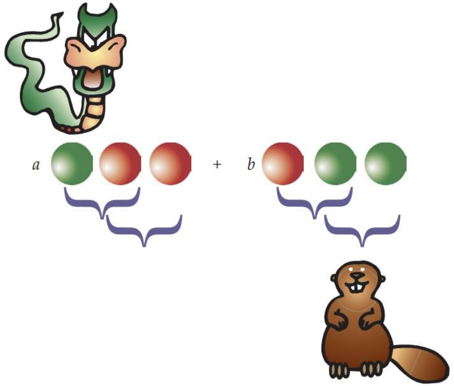
Preskill, 2025

PROBABILISTIC BIT-FLIP ERROR

$$a|000\rangle + b|111\rangle \rightarrow a|000\rangle + b|111\rangle + \varepsilon(a|000\rangle + b|111\rangle) + O(\varepsilon^2),$$

$$|\varepsilon| \ll 1.$$

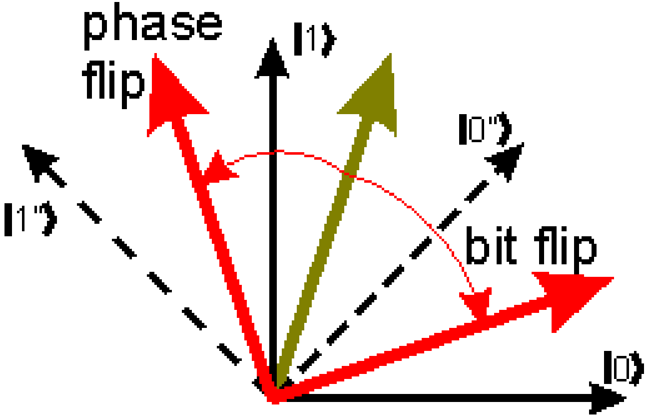
- Will measure damaged state occasionally
  - When this happens,



Preskill, 2025

CHALLENGES FOR QUANTUM ERROR CORRECTION

- ~~No-cloning theorem~~
- Different kinds of error
  - ~~Bit-flip error~~
  - Phase error
- ~~Probability continuum of errors~~
- ~~Destructive measurement~~
  - ~~$a|01\rangle + b|10\rangle$~~



[Steane, Quantum Error Correction, *an informal introduction*]

QUANTUM ERROR CORRECTION: PHASE ERRORS

PHASE ERROR

Phase error: $a\,|\bar{0}\rangle + b\,|\bar{1}\rangle \rightarrow a\,|\bar{0}\rangle - b\,|\bar{1}\rangle$

$$|0\rangle \rightarrow |\bar{0}\rangle \equiv \frac{1}{2^{3/2}}\,(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle),$$

$$|1\rangle \rightarrow |\bar{1}\rangle \equiv \frac{1}{2^{3/2}}\,(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle).$$

PHASE ERROR

General state:

$$a|\bar{0}\rangle + b|\bar{1}\rangle \equiv \frac{a}{2^{3/2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) + \frac{b}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

With phase error:

$$a|\bar{0}\rangle + b|\bar{1}\rangle \equiv \frac{a}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) + \frac{b}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

CORRECTING PHASE ERRORS

With phase error:

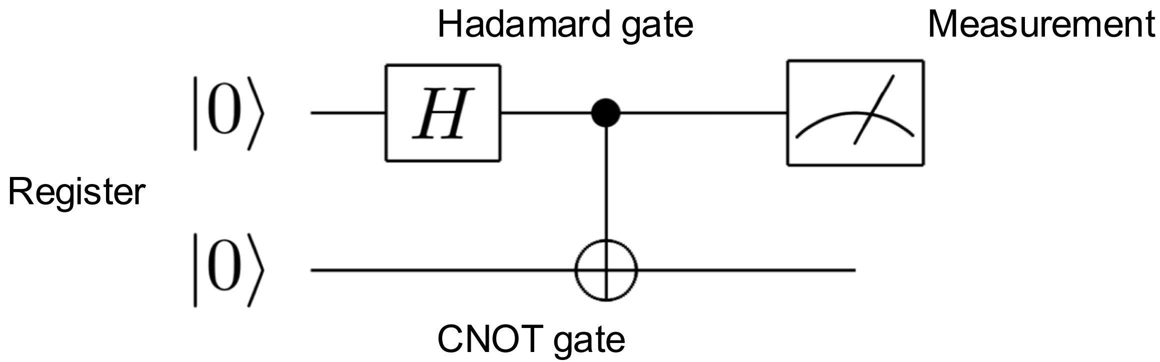
$$a\,|\bar{0}\rangle + b\,|\bar{1}\rangle \equiv \frac{a}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) + \frac{b}{2^{3/2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

1. Compare the phases of each pair of clusters
2. If one cluster differs from the other two, apply phase transformation to correct error

This is a nine-qubit code; the same can be accomplished with five qubits

COLLECTIVE MEASUREMENTS AND FAULT TOLERANCE

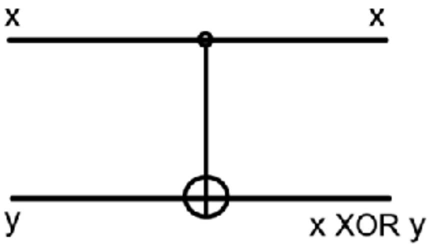
QUANTUM CIRCUIT



dojo.qulacs.org/

COLLECTIVE MEASUREMENT

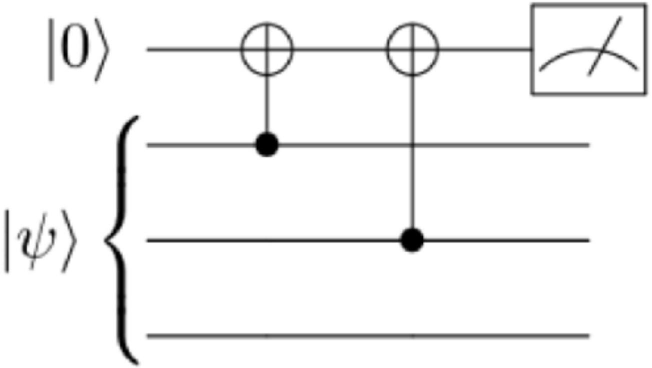
CNOT gate:



x	y	x	x XOR y
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Rajan, 2024

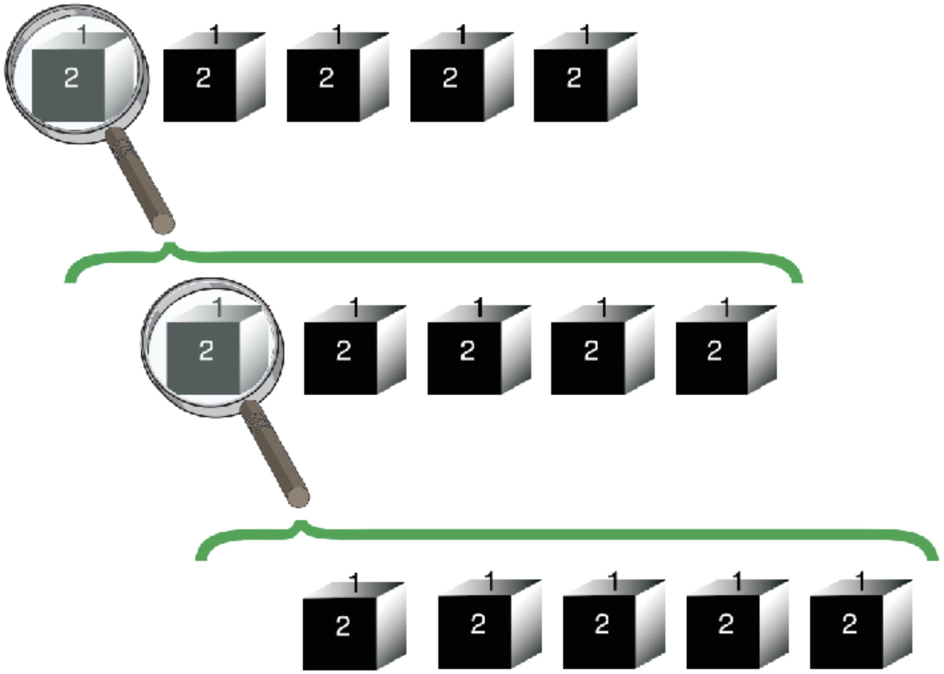
To compare two qubits:



Pesah, 2023

FAULT TOLERANCE

- More layers of qubits = more resistance to error
- Also consider gate depth: number of gates in a circuit that need to be run in series
  — Errors increase as gate depth increases

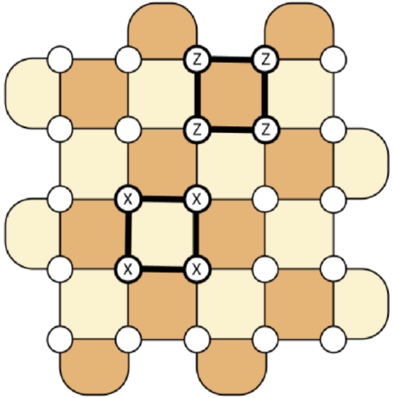


[Preskill, 2025]

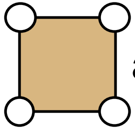
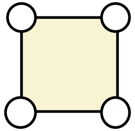
QUANTUM ERROR CORRECTION CODES: SURFACE CODES

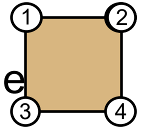
SURFACE CODE

Encodes one logical qubit with a 2D grid of physical qubits



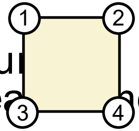
[Pesah, 2022]

Stabilizers:  and 



detects bit flips---returns $z_1 \times z_2 \times z_3 \times z_4$, where z_n is the outcome of the Pauli-Z measurement on qubit *n.*

$z_n = -1$ if a bit flip error occurred; otherwise, $z_n = 1$

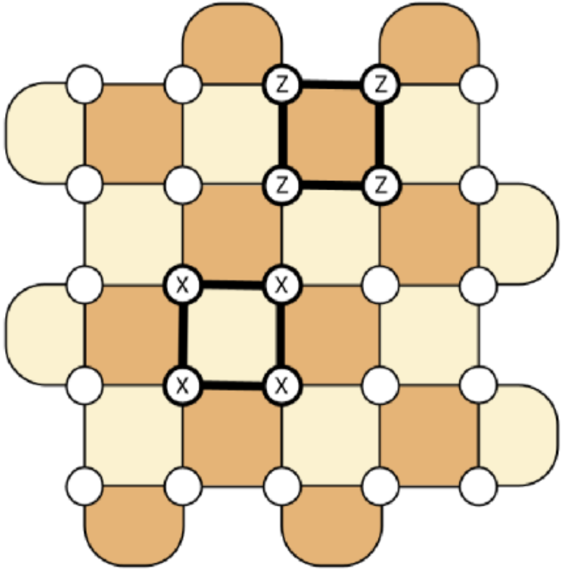


detects phase flips---returns $x_1 \times x_2 \times x_3 \times x_4$, where x_n is the outcome of the Pauli-X measurement on qubit *n.*

$x_n = -1$ if a phase flip error occurred; otherwise, $x_n = 1$.

ERROR CORRECTION WITH SURFACE CODE

1. Measure stabilizers to create *syndrome* (map of stabilizer values)
2. Decode syndrome using classical algorithm to determine most likely errors
3. Correct errors



Pesah, 2022

SURFACE CODE PROS AND CONS

Pros

- High error threshold
- Local operations
  - — Easier to implement
  - — Scalable
  - — Greater fidelity

Cons

- High overhead
  - — Estimated 1,000—10,000 qubits per logical qubit (Fowler et al., 2012)
- Can be difficult to decode

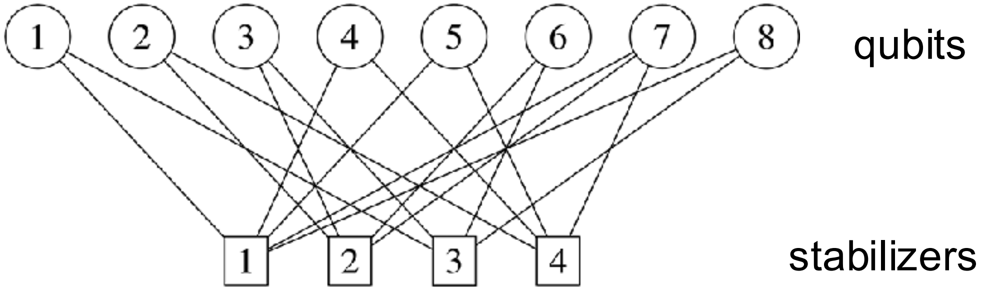
QUANTUM ERROR CORRECTION CODES: LOW-DENSITY PARITY-CHECK (LDPC) CODES

LDPC CODES

Number of qubits in each stabilizer is bounded by a constant

Number of stabilizers each qubit participates in is bounded by a constant

Represented by Tanner graphs:



And LDPC matrices:

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Amaricai and Boncalo, 2017

LDPC CODES PROS AND CONS

(for codes 'better' than surface codes)

Pros

- Smaller overhead
  - More scalable
- Better code distance (measure of code's error-correcting ability)

Cons

- Not local in 2D
- More complex to decode
- Difficult to implement in hardware

CURRENT TECHNOLOGIES + FUTURE WORK

FIDELITY AND COHERENCE TIMES

Fidelity: how accurately a gate operates

- Desired fidelity: >99.9%

Coherence time: how long a qubit remains in a state before being disrupted by the environment

- Want long coherence times
  - For surface code, estimated 1-10 μs physical qubit coherence time needed (Fowler et al., 2012)

Error correction improves fidelity and logical qubit coherence time

CURRENT TECHNOLOGIES

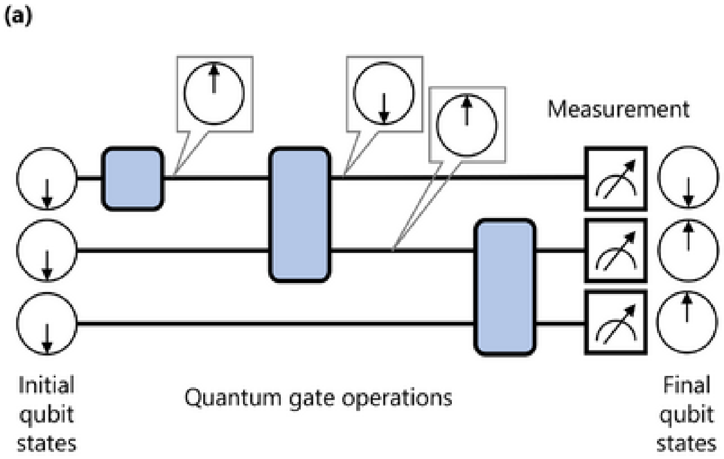
	Superconducting	Photonic	Cold Atoms	Trapped Ions	Si Quantum Dots	NV-Center	Topological
							
Companies	Google IBM amazon	XANADU PsiQuantum	PASQAL atom computing QuEra Computing Inc.	QUANTINUUM IONQ oxford ionics	intel QuTech	QUANTUM BRILLIANCE	Microsoft
Number of Physical Qubits	>1000	>100	>1000	>50	>10	>10	n/a
Coherence Time	Transmon: ~500 us Fluxonium: ~1 ms	Time of photon flight	ms – s regime	> 10 mins	ms regime	s (cryo) ms (RT)	n/a
Gate Fidelities	Single: 99.99% Two: 99.9%	One: 99.9% Two: 99.5%	Single: 99.9% Two: 99.5%	Single: > 99.99% Two: 99.97%	Single: 99.9 % Two: 99 %	Single: 99.999% Two: 99.93%	
Gate Time	10 – 100 ns	1 – 100 ns	10 ns – 1us	1 – 100 us	0.1 – 10 us	~ 100 ns	n/a
Scalability	3D packaging Uniformity Optical link to >1E4	Large scale PIC with low loss	PIC for scalable control Optical link to >1E5	PIC & IC for scalable control Optical link to >1E3	3D packaging & cryoCMOS Yield & uniformity	C13 isotope growing random Optical link to > 1E2	n/a
Pros	Highly researched Engineerable qubits Market mature	Leverage telecom industry Network scaling	Large modules No mK cryogenics High connectivity	High gate fidelity No mK cryogenics High connectivity	Using CMOS module & expertise	Long coherence Quantum networking	Inherent fault tolerant properties
Cons	mK cryogenics Wiring bottleneck Network scaling	Large overhead Photon loss errors	New technology Long preparation times	Slow gate time Small modules	Strict material requirements Wiring bottle neck	Depending on random effects Small modules	Need proof-of-concept demonstration

Source: Applied Materials, industry data

Blank, 2024

QUANTUM COMPUTING MODELS

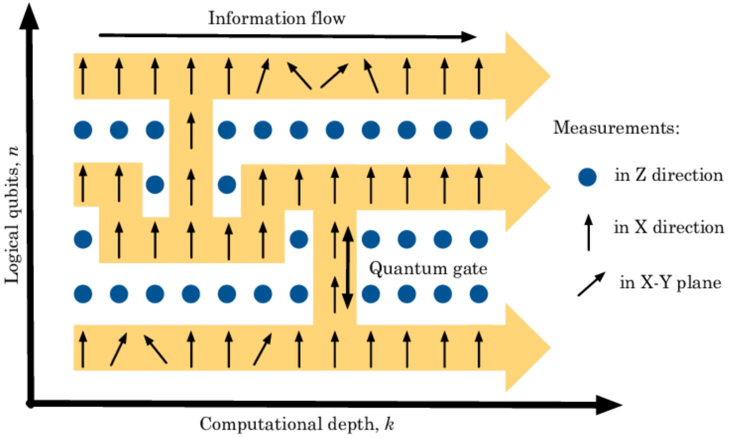
Gate-based



Chia et al., 2024

IonQ, Google, IBM, Rigetti, Quantinuum

Measurement-based

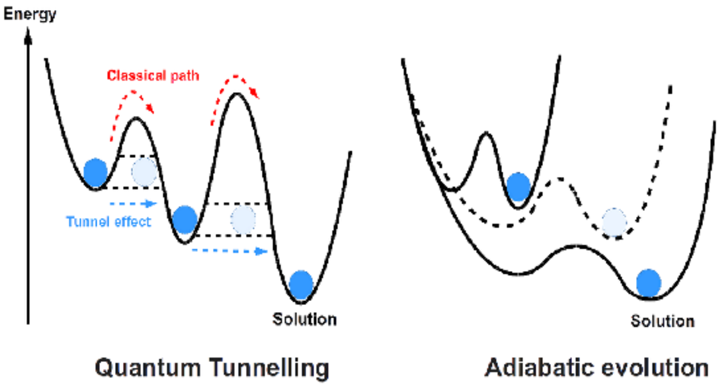


Gimeno-Segovia et al., 2015

Xanadu, PsiQuantum

QUANTUM COMPUTING MODELS, CONT.

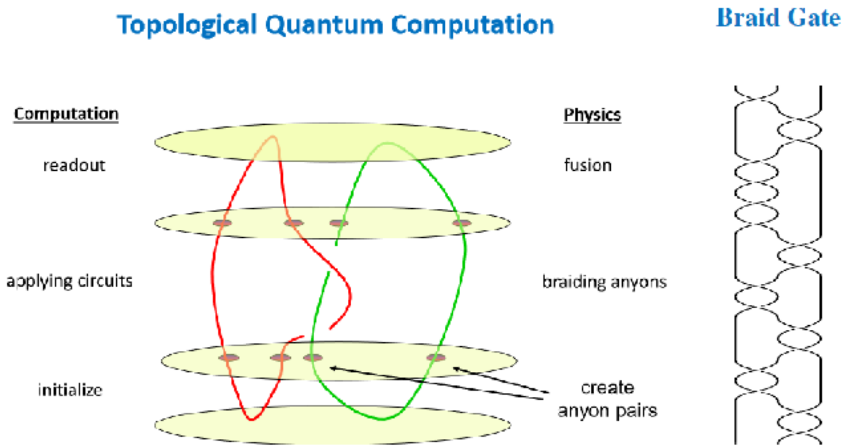
Adiabatic quantum computing, quantum annealing



Cervera-Lierta, 2018

D-Wave

Topological



ncatlab.org

Microsoft

FUTURE WORK

Error correction

- Current: a few qubits and operations
- Needed: hundreds/thousands of qubits, millions/billions of operations

Hardware

- Control, fidelity, scaling up

Algorithms

- Efficiency/quantum advantage
- Hybrid classical-quantum algorithms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