Using Structure to Solve Optimization Problems in Quantum Computing

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Problem setup

$$\underset{x}{\text{minimize}}\ f(x) = h(S(x))$$

$$\text{subject to: } x \in \mathcal{D} \subset \mathbb{R}^n$$

where the objective f depends on the output(s) from a simulation S and a known function h.

- Assume derivatives of S are not available
- The dimension n is small
- Evaluating S is expensive
- Constraints defining $\mathcal{D}$ may or may not depend on S

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- For large N, computing the QFI can be prohibitively difficult. Many papers maximize (upper) bounds of QFI

Maximizing concurrence

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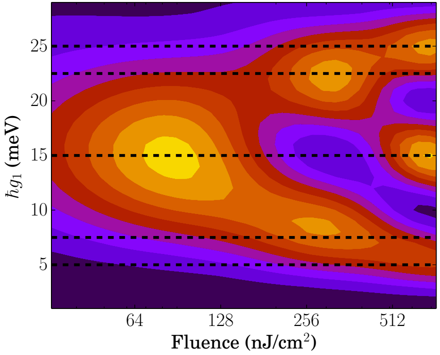

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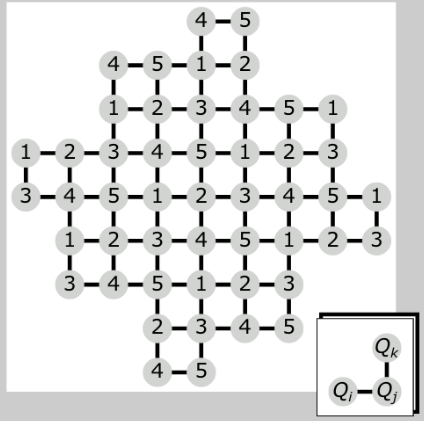
- Scaling fixed-frequency architectures requires precise relative frequency requirements.
- Want to avoid collisions in frequencies.



Hertzberg et al., https://arxiv.org/pdf/2009.00781.pdf

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$$|2f_i + \alpha_i - f_k - f_j| \geq \delta_7$$
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- A possible objective:

$$\text{maximize} \sum_i w_i \delta_i$$

with $\delta_i \geq \bar{\delta}_i$.

Two solutions on 6-node ring